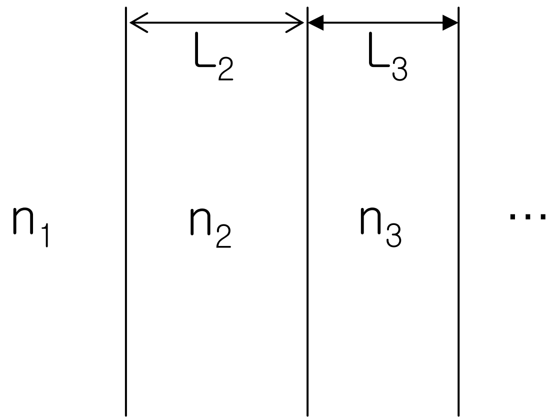
Lect. 7: Multiple Dielectric Interface



Complex Problem:
Requires an advanced technique

Consider two special cases:

$$L_i = m\frac{\lambda}{2n_i} \text{ (Half Wavelength) or } L_i = (m+\frac{1}{2})\frac{\lambda}{2n_i} \text{ (Quarter Wavelength)}$$

Model it with transmission lines

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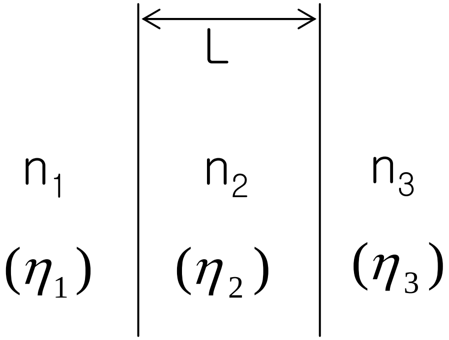
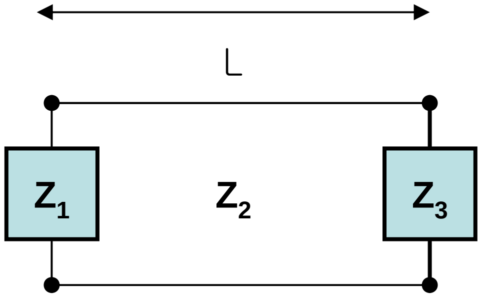
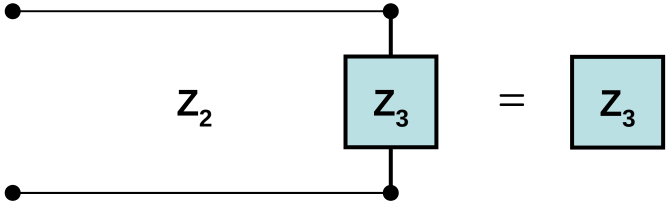
Model dielectric layers with Transmission Lines





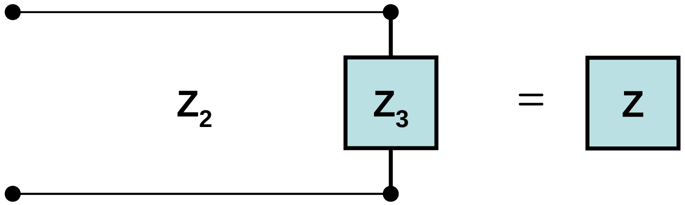
If $L = \frac{\lambda}{2}$

(One rotation on Smith Chart!)



If $L = \frac{\lambda}{4}$

(Half rotation on Smith Chart!)



$$Z = Z_2 \frac{1}{Z_3 / Z_2} = \frac{Z_2^{\,2}}{Z_3}$$

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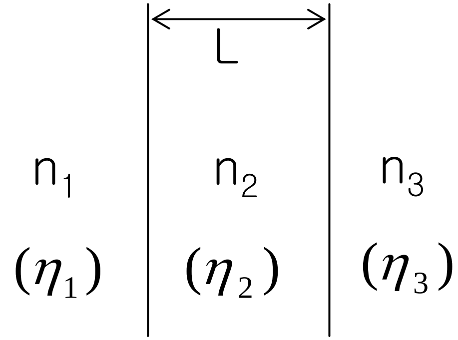

$L = \dfrac{\lambda}{2n_2}$

n_1 (η_1) | n_3 (η_3)

$$\therefore\ r = \frac{n_1 - n_3}{n_1 + n_3},\ t = \frac{2n_1}{n_1 + n_3}$$

$L = \dfrac{\lambda}{4n_2}$

n_1 (η_1) | $n = \dfrac{n_2^{\ 2}}{n_3}$ $\left(\eta = \dfrac{\eta_2^{\ 2}}{\eta_3}\right)$

$$r = \frac{n_1 - n_2^{\ 2}/n_3}{n_1 + n_2^{\ 2}/n_3},\ t = \frac{2n_1}{n_1 + n_2^{\ 2}/n_3}$$

$$\therefore\ r = \frac{n_1 n_3 - n_2^{\ 2}}{n_1 n_3 + n_2^{\ 2}},\ t = \frac{2n_1 n_3}{n_1 n_3 + n_2^{\ 2}}$$

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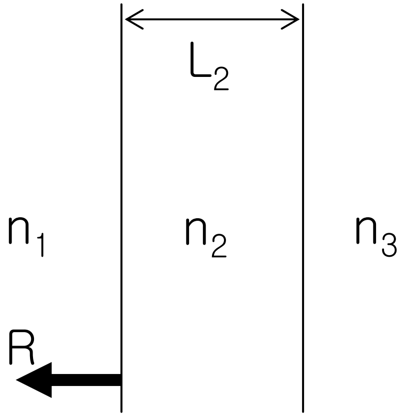
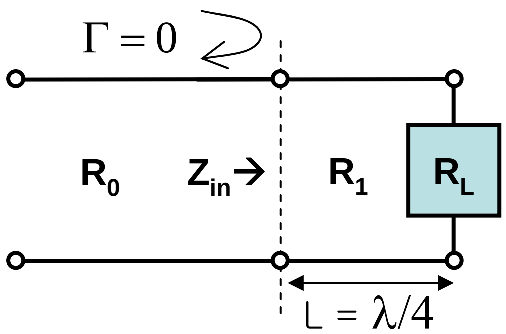
Anti-Reflection coating: Determine L_2 and n_2 so that R=0



With $L = \dfrac{\lambda}{4n_2}$ n1 | $n = \dfrac{{n_2}^2}{n_3}$

Since $r = \dfrac{n_1 n_3 - {n_2}^2}{n_1 n_3 + {n_2}^2}$,

$n_1 n_3 - {n_2}^2 = 0$ or $n_2 = \sqrt{n_1 n_3}$ for $R = 0$



$\therefore R_1 = \sqrt{R_0 R_L}$

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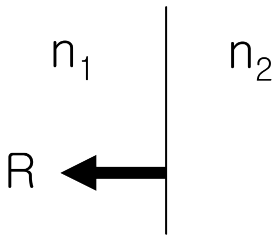
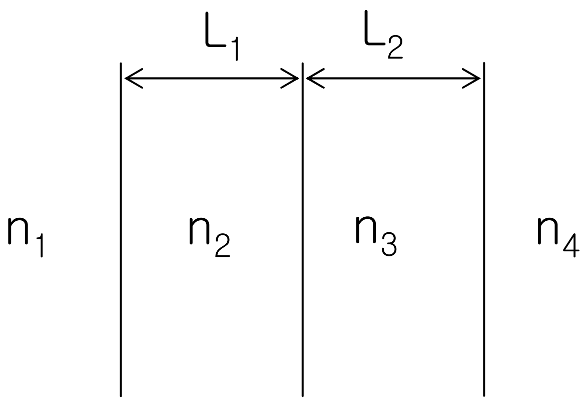
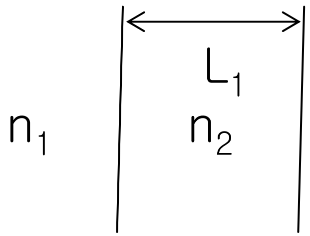
High-Reflection Coating



$$R = \left(\frac{n_1 - n_2}{n_1 + n_2}\right)^2 \quad R \to 1 \text{ if } n_1 >> n_2 \text{ or } n_1 << n_2$$

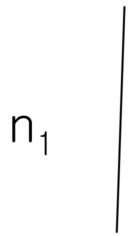


$$\text{With } L_2 = \frac{\lambda}{4n_3}$$



$$n = \frac{n_3^{\ 2}}{n_4}$$

$$\text{With } L_1 = \frac{\lambda}{4n_2}$$



$$\frac{n_2^{\ 2}}{n_3^{\ 2}/n_4}$$

$$R = \left(\frac{n_1 - \left(\frac{n_2}{n_3}\right)^2 n_4}{n_1 + \left(\frac{n_2}{n_3}\right)^2 n_4} \right)^2$$

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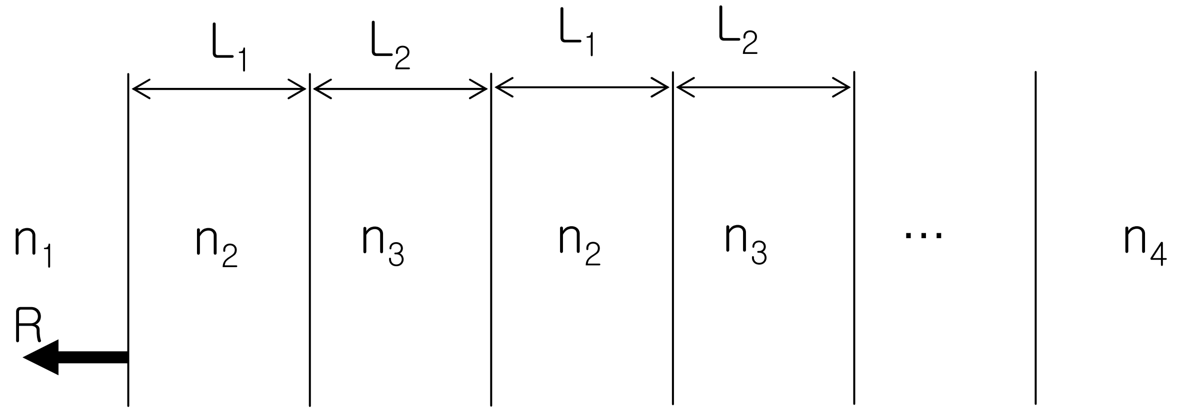
Higher-Reflection Coating

Repeat the quarter-wavelength pair m times.



$$R = \left(\frac{n_1 - \left(\frac{n_2}{n_3}\right)^{2m} n_4}{n_1 + \left(\frac{n_2}{n_3}\right)^{2m} n_4} \right)^2$$

$$\text{If } n_2 > n_3,\ R \sim \left(\frac{-(n_2/n_3)^{2m} n_4}{+(n_2/n_3)^{2m} n_4} \right)^2 = 1$$

$$\text{If } n_2 < n_3, R \sim \left(\frac{n_1}{n_1} \right)^2 = 1$$

=> Dielectric mirror

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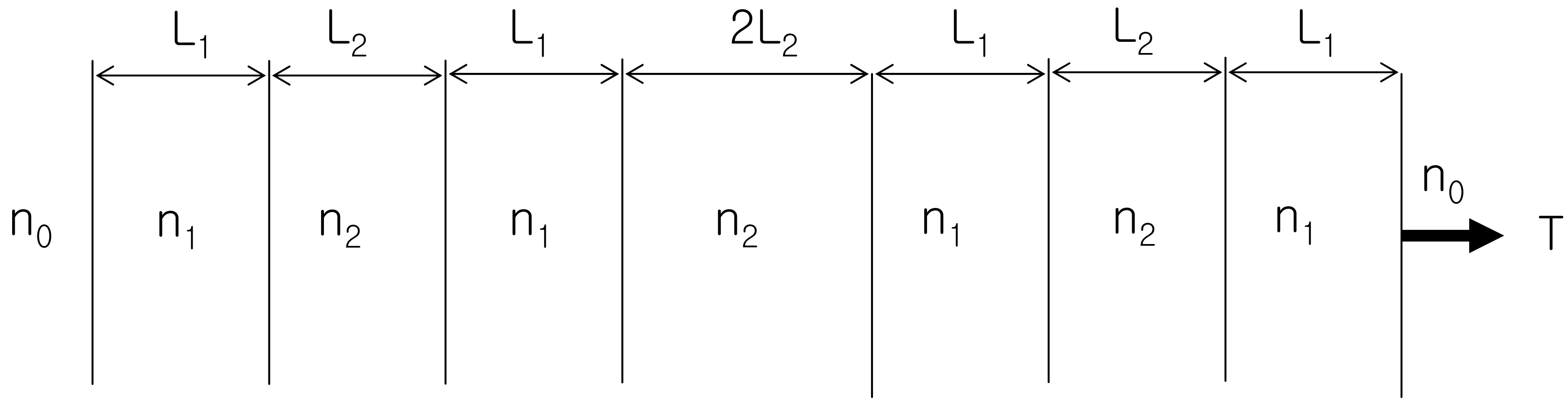
Determine T for the following multiple dielectric layers.



$$n_1 = 1.35,\ n_2 = 2.3,\ L_{1,2} = \frac{\lambda}{4n_{1,2}}$$